Example : nonlinear heat conduction in 1D

$$\begin{cases} N(d) = F^{ext} \\ \text{where} \quad N(d) = \mathop{\mathrm{A}}_{e=1}^{n_{el}} n^e(d^e), \quad F^{ext} = \mathop{\mathrm{A}}_{e=1}^{n_{el}} f^e. \end{cases}$$

$$n_a^e(d^e) = \int_{\Omega^e} N_{a,x}\, \kappa(u^h)\, u^h_{,x}\, dx$$

$$u^h = \sum_{b=1}^{n_{en}} N_b d_b^e \qquad u^h_{,x} = \sum_{b=1}^{n_{en}} N_{b,x} d_b^e$$

$$f_a^e = \int_{\Omega^e} N_a f \, d\Omega + \int_{\Gamma_h^e} N_a h \, d\Gamma$$

if κ is const, $$n_a^e(d^e) = \sum_{b=1}^{n_{en}} \underbrace{\int_{\Omega^e} N_{a,x}\, \kappa\, N_{b,x}\, dx}_{\text{element stiffness matrix } K_{ab}^e} d_b^e$$

if $\kappa = \kappa(u^h)$, we need to assemble the consistent tangent

$$DN(d) = \mathop{\mathrm{A}}_{e=1}^{n_{en}} Dn^e(d^e)$$

$$Dn^e(d^e) = \left[\frac{\partial n_a^e}{\partial d_b^e} \right] \qquad 1 \leq a, b \leq n_{en}$$

user need to provide this function.

$$\frac{\partial n_a^e}{\partial d_b^e} = \int_{\Omega^e} N_{a,x} \frac{\partial \kappa}{\partial u}(u^h)\, N_b\, u^h_{,x} + N_{a,x}\, \kappa(u^h)\, N_{b,x}\, dx$$

In the evaluation of u^h,

$$d_a^e = \begin{cases} d_P & P = LM(a,e) \neq 0 \\ g(x_A) & P = LM(a,e) = 0 \quad A = IEN(a,e) \end{cases}$$

$\Rightarrow$ the (M) problem is $\quad DN(d^{(i)})\,\Delta d^{(i)} = F^{ext} - N(d^{(i)})$

consists solely of the unknown dofs.

the Dirichlet dofs enter as a homogeneous essential BC. for the incremental Δd.

Remark 1: $DN = K$ is non-symmetric in this case.

solver technology.

Remark 2: In certain cases, the external force depends on u^h (e.g. traction prescribed on the deformed configuration), One will have to perform linearization for $F^{ext}(d)$ as well.