Nonlinear heat conduction.

(S) Given the heat source f, temperature boundary cond. g, heat flux boundary condition h, determine $u : \bar{\Omega} \to \mathbb{R}$ s.t.

$$q_{i,i} = f \quad \text{in } \Omega$$

$$u = g \quad \text{on } \Gamma_g$$

$$-q_i n_i = h \quad \text{on } \Gamma_h$$

where the constitutive law is

$$q_i = -\kappa_{ij}(x, u)\, u_{,j}$$

generalized Fourier's law.

(W) Given f, h, g as in (S), find $u \in \mathcal{S}$ st. $\forall\, w \in \mathcal{V}$

$$a(w, u) = (w, f) + (w, h)_{\Gamma_h}$$

in which

$$a(w, u) := \int_\Omega w_{,i}\, \kappa_{ij}(u)\, u_{,j}\, d\Omega$$

$$(w, f) := \int_\Omega w f\, d\Omega$$

$$(w, h)_{\Gamma_h} := \int_{\Gamma_h} w h\, d\Gamma$$

(G) Given f, g, h as in (S), find $u^h \in \mathcal{S}^h$ s.t. for $\forall\, w^h \in \mathcal{V}^h$

$$a(w^h, u^h) = (w^h, f) + (w^h, h)_{\Gamma_h}$$

same as in (W)

restate the (W) with $\mathcal{S}^h \subset \mathcal{S}$ $\mathcal{V}^h \subset \mathcal{V}$.

previously was Kd in linear problems.

(N) Determine d from $N(d) = F$, wherein

$$N(d) = \mathop{\mathbb{A}}_{e=1}^{n_{el}} n^e(d^e) \qquad F = \mathop{\mathbb{A}}_{e=1}^{n_{el}} f^e.$$

nonlinear algebraic equations

external heat supply from f & h.

$$n^e = \{ n^e_a \}_{a=1}^{n_{en}} \qquad n^e_a = \int_{\Omega^e} N_{a,i}\, \kappa_{ij}(u^h)\, u^h_{,j}\, d\Omega$$

$$u^h = \sum_{b=1}^{n_{en}} N_b\, d^e_b$$

$$u^h_{,j} = \sum_{b=1}^{n_{en}} N_{b,j}\, d^e_b$$

Remark: Here, a is no more linear w.r.t. u^h. We cannot have the split of $a(w^h, v^h + g^h)$. We will postpone our treatment of g-BC until (N) is linearized.