

Nonlinear heat conduction.

(S) { Given the heat source f , temperature boundary cond. g .
 heat flux boundary condition h , ~~and~~ determine $u: \bar{\Omega} \rightarrow \mathbb{R}$
 s.t.

$$\begin{aligned} \mathcal{L}i.i &= f && \text{in } \Omega \\ u &= g && \text{on } \Gamma_g \\ -q_i n_i &= h && \text{on } \Gamma_h \end{aligned}$$

where the constitutive law is

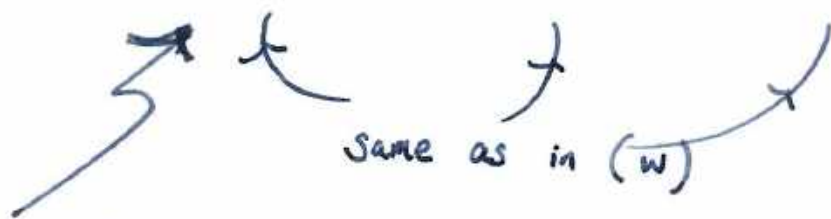
$$\mathcal{L}i = -\kappa_{ij}(x, u) u_{,j} \quad \leftarrow \text{generalized Fourier's law.}$$

(W) { Given f, h, g as in (S), find $u \in \mathcal{S}$ st. $\forall w \in \mathcal{V}$
 $a(w, u) = (w, f) + (w, h)_{\Gamma_h}$
 in which

$$\begin{aligned} a(w, u) &:= \int_{\Omega} w_{,i} \kappa_{ij}(u) u_{,j} \, d\Omega \\ (w, f) &:= \int_{\Omega} w f \, d\Omega \\ (w, h)_{\Gamma_h} &:= \int_{\Gamma_h} w h \, d\gamma \end{aligned}$$

(G) Given f, g, h as in (S), find $u^h \in \mathcal{S}^h$ s.t.
 for $\forall w^h \in \mathcal{V}^h$

$$a(w^h, u^h) = (w^h, f) + (w^h, \gamma_h)_{\Gamma_h}$$



same as in (w)

restate the (w) with $\mathcal{S}^h \subset \mathcal{S}$ $\mathcal{V}^h \subset \mathcal{V}$.

(N) Determine d from $N(d) = F$, wherein \leftarrow previously was Kd in linear problems.

$$N(d) = \sum_{e=1}^{nel} A^e n^e(d^e) \quad F = \sum_{e=1}^{nel} f^e$$

nonlinear algebraic equations

$$n^e = \{ n_a^e \}_{a=1}^{nel}$$

external heat supply from f & h .

$$n_a^e = \int_{\Omega_a^e} N_{a,i} x_{ij}(u^h) u_{,j}^h d\Omega$$

Remark: Here, a is no more linear w.r.t. u^h . We cannot have the split of $a(w^h, v^h + g^h)$, so we will postpone our treatment of g -BC until (N) is linearized.

$$u^k = \sum_{b=1}^{nel} N_b d_b^e$$

$$u_{,j}^k = \sum_{b=1}^{nel} N_{b,j} d_b^e$$