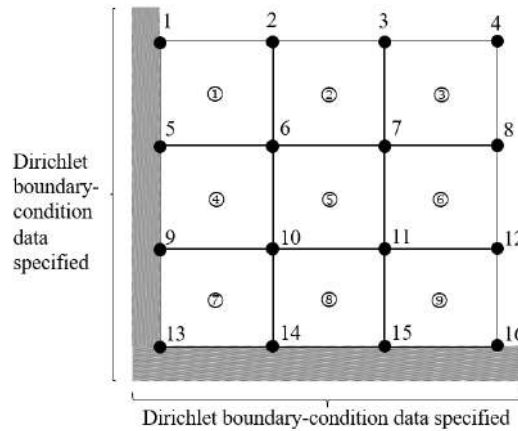


**Homework 2**  
**Due: Mar. 25 2024**

1. Consider the following mesh. Set up the ID, IEN, and LM arrays.



2. Determine  $\tilde{\xi}_l$ ,  $\tilde{\eta}_l$ ,  $\tilde{\zeta}_l$ , and  $W_l$  for the one-point and  $2 \times 2 \times 2$  Gaussian rules in three dimensions. You will have to first determine Gaussian rules in one dimension by consulting the Hughes book pp. 141-142.

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 g(\xi, \eta, \zeta) d\xi d\eta d\zeta \cong \sum_{l^{(1)}=1}^{n_{int}^{(1)}} \sum_{l^{(2)}=1}^{n_{int}^{(2)}} \sum_{l^{(3)}=1}^{n_{int}^{(3)}} g(\tilde{\xi}_{l^{(1)}}^{(1)}, \tilde{\eta}_{l^{(2)}}^{(2)}, \tilde{\zeta}_{l^{(3)}}^{(3)}) W_{l^{(1)}}^{(1)} W_{l^{(2)}}^{(2)} W_{l^{(3)}}^{(3)}$$

$$= \sum_{l=1}^{n_{int}} g(\tilde{\xi}_l, \tilde{\eta}_l, \tilde{\zeta}_l) W_l$$

3. Clone the code in Chapter-01 from <https://github.com/M3C-Lab/FEM-1D-demo>

- a. Read the code in the driver.m and draw a flowchart of it.
- b. The driver.m code computes the relative error

$$e = \frac{\left( \int_0^1 (u^h - u)^2 dx \right)^{1/2}}{\left( \int_0^1 u^2 dx \right)^{1/2}}$$

Run the code with 2, 4, 6, 8, 10, 12, 14, 16 elements with uniform meshsize. Plot the relative error against the meshsize in the log-log plot (i.e., plot  $\log(e)$  against  $\log(h)$ ). Determine the slope of the graph.

- c. Run the code with higher-order element by changing the value of pp. Re-do the problem b for  $pp = 2, 3, 4, 5, 6$ .
- d. Read Hughes book 1.11. Make sure you understand the Gauss Elimination. Use the command `lu` in Matlab to do the LU-factorization of the stiffness matrix and solve the matrix problem. Experiment it and compare it with the `\` operator used in the code. You may want to consult the following matlab page for references.

<https://www.mathworks.com/help/matlab/ref/lu.html>

- e. Matlab also provides iterative methods for solving linear systems. Learn the usage of the command `gmres` in Matlab. You may set `restart=maxit=10000`. Set the value of `tol` to be  $10^{-2}$ ,  $10^{-4}$ , and  $10^{-6}$  and compare the solutions against those obtained by the direct method (i.e. LU-factorization). Give your comments. You may need to consult the following page for references.  
<https://www.mathworks.com/help/matlab/ref/gmres.html>
- f. The equivalence between (G) and (M) is contingent upon the accuracy of the quadrature. The effect of numerical quadrature is discussed in Hughes book p. 191. Run the code with the element of degree six (i.e. set `pp=6` in the driver). Experiment the code with 1,2,3,4,5,6 quadrature points, respectively. Report your observation and make comments.

4. Read Hughes book Appendix 3.1, pp. 164-174. Consider the quadratic tetrahedral element (Figure 3.1.9). Suppose the physical element is constructed by the *isoparametric* approach. Discuss in detail the steps for calculating the shape function derivatives and the element stiffness matrix.